NSTX Upgrade

OH Conductor Fatigue and Fracture Mechanics Analyses

NSTXU-CALC-133-09-00

Rev 0

Nov 2010

Prepared By:

Peter Titus Digitally signed by Peter Titus
DN: cn=Peter Titus, o, ou,
email=ptitus@pppl.gov, c=US
Date: 2011.02.23 11:56:21 -05'00'

Peter Titus, PPPL Mechanical Engineering

Reviewed By:

Irving Zatz Digitally signed by Irving Zatz
DN: cn=Irving Zatz, o=PPPL, ou,
email=zatz@pppl.gov, c=US
Date: 2011.02.23 17:07:03 -05'00'

Irving Zatz Engineering Analysis Division

Jim Chrzanowski Digitally signed by Jim Chrzanowski
DN: cn=Jim Chrzanowski, o=PPPL,
ou=Enginering, email=jchrzano@pppl.gov, c=US
Date: 2011.02.28 08:26:32 -05'00'

James Chrzanowski NSTX Cognizant Engineer

PPPL Calculation Form

Calculation # NSTXU-CALC-133-09-00 **Revision #** **00** __ **WP #, 1672** **(ENG-032)**

Purpose of Calculation: **(Define why the calculation is being performed.)**

To establish a fatigue allowable for the OH coil conductor planned for use in the NSTX upgrade

References **(List any source of design information including computer program titles and revision levels.)**

[1] OH Stress Analysis, A. Zolfaghari, Calc #NSTXU-CALC-133-08
[2] Memo: Fatigue life of VS coil made of pure copper C11000 To:Peter Titus From:Jun Feng Date: 12/21/2009
[3] Memo to Charlie Neumeyer, NSTX distribution From: Peter Titus, Jun Feng Subject: Fatigue Analysis of OH Conductor Date: November 24 2009
[4] NATIONAL SPHERICAL TORUS EXPERIMENT CENTER STACK RESEARCH AND DEVELOPMENT FINAL REPORT No. 13-970430-JHC Prepared By: **James H. Chrzanowski April 30, 1997** PRINCETON UNIVERSITY PLASMA PHYSICS LABORATORY
[5] ITER In-Vessel Coil Memo: "Fatigue life of VS coil made of pure copper C11000 Memo" To:Peter Titus From:Jun Feng Date: 12/21/2009, Included in Appendix B

Table No. 4-11
FATIGUE TEST RESULTS-OH TYPE VII JOINT

Assumptions **(Identify all assumptions made as part of this calculation.)**

The fracture mechanics calculations have been performed for three crack areas: .125,.25 and .5 mm^2 which are taken to correspond to crack depths of .353, .5, and .7 mm. The ratio a/b or crack depth to width is taken as 1.0

Calculation **(Calculation is either documented here or attached)**

See the Body of the calculation

Conclusion **(Specify whether or not the purpose of the calculation was accomplished.)**

Hoop Stress, or max principal stress peak in the OH conductor must remain below 125 MPa to satisfy fracture based fatigue requirements.

Cognizant Engineer's printed name, signature, and date

James Chrzanowski Jim Chrzanowski Digitally signed by Jim Chrzanowski DN: cn=Jim Chrzanowski, o=PPPL, ou=Enginering, email=jchrzano@pppl.gov, c=US Date: 2011.02.28 08:26:54 -05'00'

I have reviewed this calculation and, to my professional satisfaction, it is properly performed and correct.

Checker's printed name, signature, and date

Irving Zatz Irving Zatz Digitally signed by Irving Zatz DN: cn=Irving Zatz, o=PPPL, ou, email=zatz@pppl.gov, c=US Date: 2011.02.28 10:32:57 -05'00'

Table of Contents

Executive Summary:

The OH coil was originally sized based on static allowables. Two areas were checked - the peak ID Tresca stress, which must be below 1.5*Sm, and the average stress in the cross section, which must be below Sm. These evaluations have been carried out in the OH coil stress calculation, ref [1].

NSTX structural criteria and the GRD require fatigue to be addressed. The criteria allows either SN or fracture mechanics evaluations of fatigue. For SN evaluations, the more restrictive of 2 on stress and 20 on life must be met. For the fracture mechanics evaluation, a factor of 2 on flaw size, 1.5 on fracture toughness, and 2 on life must be met. The stress levels in the NSTX-U OH coil satisfy the fracture mechanics criteria, and therefore satisfy the NSTX structural requirements.

Criteria	Stress Level ant Type	Actual ref [1]	
SN 2 on stress	112 MPa (Tresca)	142	Fails
SN 20 on life	180 (Tresca)	142	Passes
Fracture Mechanics with a flaw size less than .7mm 1.5 on KIc and 2 on Cycles	140 MPa (Max Principal or Hoop)	101	Passes
4 on cycles	125 MPa (Max Principal or Hoop)	101	Passes

The fracture mechanics calculation forms the basis of the qualification of the OH stresses and potentially other copper conductors used in the PF system. A lower bound on the fracture mechanics results and other data is used to develop an allowable stress. Flaw sizes are assumed at this point, but will have to be imposed as an inspection requirement for the OH conductor manufacturer. Measured NSTX OH conductor braze joint fatigue life is included in the evaluation, as well as published SN data for comparison.

The fracture mechanics calculations have been performed for three crack areas: .125,.25 and .5 mm^2 which are taken to correspond to crack depths of .353, .5, and .7 mm. The ratio a/b or crack depth to width is taken as 1.0

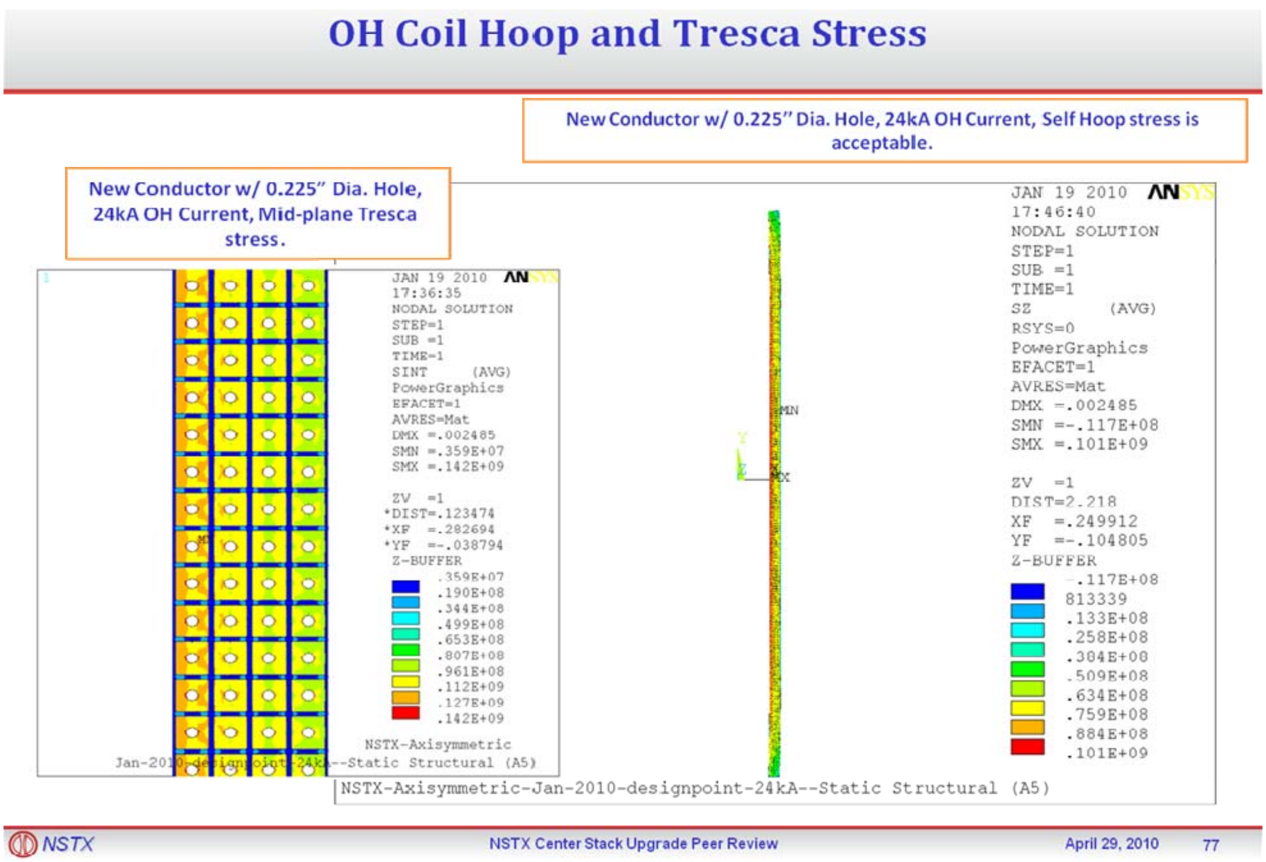


Figure 1 Stress Results from Ref [1] presented at the PDR

Digital Coil Protection System (DCPS) Input

Input to the DCPS will be developed in the OH stress calculation, and in other calculations using similar copper conductors such as the coax cable calculation . The max principal stress in the conductor must be kept below 125 MPa.

Criteria – Static Allowables for Coil Copper Stresses

The TF copper ultimate is 39,000 psi or 270 MPa . The yield is 38ksi (262 MPa). Sm is 2/3 yield or 25.3ksi or 173 MPa – for adequate ductility, which is the case with this copper which has a minimum of 24% elongation. Note that the ½ ultimate is not invoked for the conductor (It is for other structural materials) . These stresses should be further reduced to consider the effects of operation at 100C. This effect is estimated to be 10% so the Sm value is 156 MPa.

- From: I-4.1.1 Design Tresca Stress Values (Sm), NSTX_DesCrit_IZ_080103.doc
- • (a) For conventional (i.e., non-superconducting) conductor materials, the design Tresca stress values (Sm) shall be 2/3 of the specified minimum yield strength at temperature, for materials where sufficient ductility is demonstrated (see Section I-4.1.2). *
- It is expected that the CS would be a similar hardness to the TF so that it could be wound readily. For the stress gradient in a solenoid, the bending allowable is used. The bending allowable is 1.5*156 or 233MPa,

Criteria – Fatigue Allowables for Coil Copper Stresses

From the NSTX_DesCrit_IZ_080103.doc:

A fatigue strength evaluation is required for those NSTX CSU components with undetectable flaws that are either cycled over 10,000 times or are exposed to cyclic peak stresses exceeding yield stress.

From the NSTX GRD:

For engineering purposes, number of NSTX pulses, after implementing the Center Stack Upgrade, shall be assumed to consist of a total of ~ 60,000 pulses based on the GRD specified pulse spectrum.

Criteria Document
Mean Stress Effect:

$$Seq = \frac{Salt}{1 - (Smean/Su)}$$

where Su = tensile strength

The NSTX criteria document requires either a SN fatigue qualification or a fracture mechanics qualification. The SN qualification requires use of the tresca to enter the SN curve with factors of safety based on the worst of 2 x Stress or 20 on Life. The design stress in the OH is well beyond what can be qualified. The alternative is to use fracture mechanics and to implement appropriate NDE on the conductor manufacture to ensure flaw sizes are acceptable for the required life.

SECTION I-4.2.3 CRACK GROWTH LIMITATION

The following commentary and interpretation and numerical example is offered pertaining to the NSTX Design Criteria Document's discussion of Crack Growth Limitations:

- A maximum permissible initial flaw in any component, for a given specified load and environmental condition, shall be determined either analytically, in which case the initial flaw size would be backcalculated assuming four (4) times the number of design life cycles, or experimentally, based on appropriate component testing, where the initial flaw size would be based on twice the number of cycles to failure of the test article.

I-4.2.3.1 Stress Analysis

Fatigue crack growth (stage 2) is controlled primarily by maximum principal stresses (or strains). Fatigue cracks will usually propagate in the direction normal to a uniaxially applied load and the rate and direction of crack growth can be affected by loads and restraints in other directions as well as environmental conditions.

I-4.2.3.2 Material Inspection Requirement

For inspection, a back calculated initial flaw size, based on a failure scenario, cannot be smaller than twice the minimum flaw that can be resolved by nondestructive testing of the same material in a comparable geometry. The inspection procedure and results shall be included in the design documentation, along with the description of any calibration fixtures used.

An established LEFM methodology shall be used to account for the mean stress effect on crack growth rates, where deemed appropriate. The effects of closure and interaction for applicable load scenarios and values of R shall be considered.

Fatigue Data

Fracture and Deformation Div., NBS, Boulder, CO
4-15-88 N.J.S., R.P.R

CRYOGENIC PROPERTIES OF COPPER AND COPPER ALLOYS

MATERIAL C10100, C10200, C10700 COPPER (COLD-WORKED)	PROPERTY STRESS-CONTROLLED AXIAL FATIGUE LIFE; AIR, 295 K

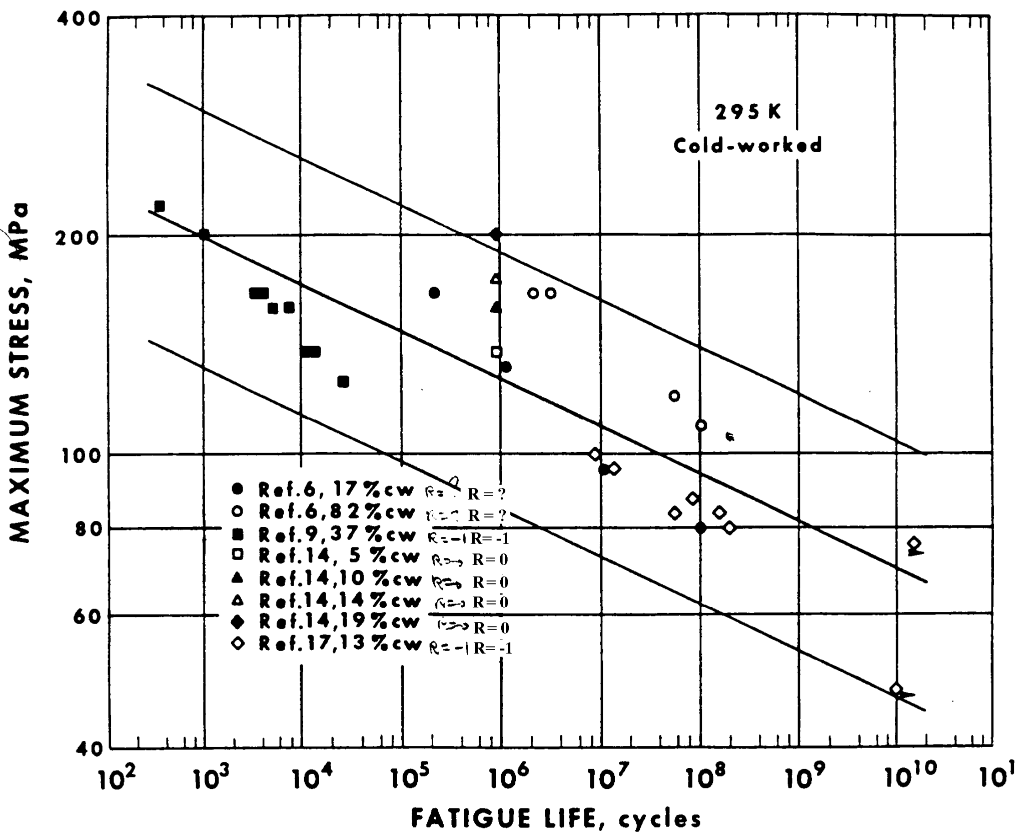


Figure 3. The data shown were used to compute the regression of maximum stress upon the number of cycles to failure [Equation (2)]. All data are presented in Table 2. Tests discontinued before failure are marked by an arrow. Product forms include bar and sheet. The percent of cold work refers to reduction of area or thickness. The R ratios are discussed in the text.

From reference [4]:

NATIONAL SPHERICAL TORUS EXPERIMENT
CENTER STACK
RESEARCH AND DEVELOPMENT
FINAL REPORT
No. 13-970430-JHC
Prepared By: **James H. Chrzanowski**
April 30, 1997
PRINCETON UNIVERSITY
PLASMA PHYSICS LABORATORY

Table No. 4-11
FATIGUE TEST RESULTS-OH TYPE VII JOINT

Note: Joints were restrained with side clamps (loosely held) to minimize moment in joint.

Specimen ID No.	Conductor Area (in^2)*	Conductor Loading (psi.)	Cyclic Loading (Lbs.)	Completed Cycles ~	Location of Failure
E	**0.184**	**20,000**	**350-3680**	**302,100**	**In conductor away from joint**
F	**0.1845**	**20,000**	**350-3680**	**417,980**	**In conductor away from joint**
G	**0.1844**	**20,000**	**350-3688**	**555,730**	**In conductor away from joint**

* **Measured prior to start of cyclic test**

From section 4.1, Specimen Preparations, of the NSTX report

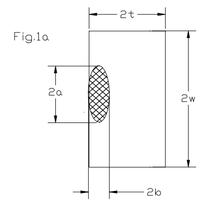
Test specimens were manufactured using both ETP and CDA 104 copper bar. Each conductor was cleaned and degreased with no additional surface preparation. The following section describes the various joint designs and the results from the static and fatigue tests.

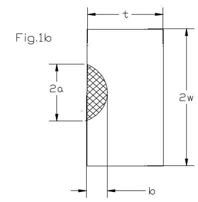
PDR Fracture Mechanics Evaluation, and Procedure:

These calculations were done based on some informal communications with Jun Feng and documented in a memo [3]. This formed the basis for subsequent calculations. Current calculations reference Jun's ITER in-vessel coil calculation which has better Paris parameters. The ITER memo is included as an appendix to this calculation.

Conductor Fracture Mechanics Evaluation

The fatigue life is obtained by integrating the Paris law using 2 point integration method:





$$da/dN = C(\Delta K)^m$$

where: da/dN is the fatigue crack growth rate (m/cycle), C and m are Paris parameters, is the stress intensity factor range at crack tip ( ). The mean stress effect is accounted by

,

$$\Delta K_{eff} = K_{\max}(1-R)^n$$

where: *n* is Walker exponent. and R is load ratio defined by . $K_{\min}/K_{\max}$

Miner's rule is applied to evaluate the accumulative damage due to multiple stress cycles during each operation cycle: [6]

$$\sum \frac{n_i}{N_i} = 1$$

where *Ni* is the number of cycles to failure at *ith* stress, *ni* is the number of cycles for *ith* stress during whole machine life.

Material

Hardened copper; Paris parameter: C=1.52e-12 m/cycles, m=4.347 ; Fracture toughness : $K_{1c} = 150 MPa\sqrt{m}$; Walker's coef: 0.8.

Reference needed for Paris constants and fracture toughness

Sample geometry

Width: 30mm (assumed) Thickness: 7.7mm

Load history

0 to 149 MPa along axial direction. Stress gradient at the hole edge is neglected.

Crack configuration

Surface crack at the edge of the hole; Initial crack dimension: 0.25mm^2, 0.5mm^2; Initial aspect ratio: 1.

Safety factors:

On crack size: 2; On fracture toughness: 1.5.

Results of fatigue crack growth life

Safety factor	Initial crack size (mm^2)	
	0.25	0.5
Safety Fact Not Applied	701,000	446,000
Safety Factor Applied	446,000	277,000

Titus CDR Calcs (Jun's Program) .5mm^2 crack area
.707mm crack x 2= .00144 m crack
with Safety Factor:
145 MPa 201244 cycles
175 MPa 103416 cycles

Analysis Results

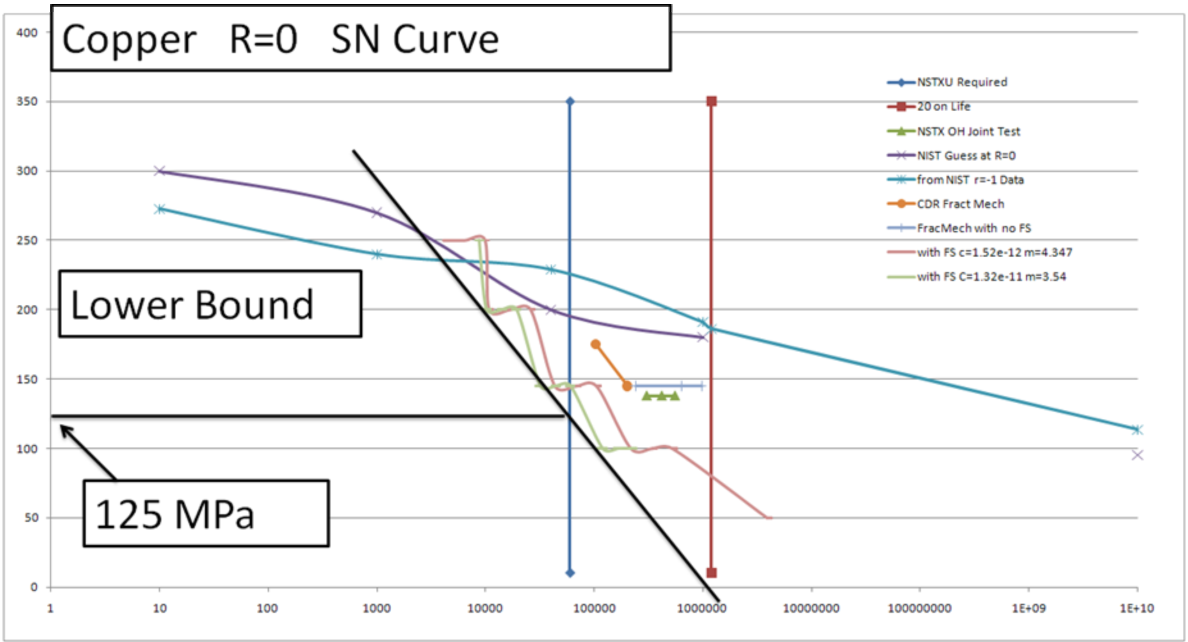


Figure 2 SN and Fracture Mechanics Fatigue Life

This is a compilation of copper R=0 fatigue data The vertical lines are at 60,000 cycles and 1200000 cycles which represents 20 times the required 60,000 pulses as specified in the GRD. The fit line represents the lower bound of the data. The plot includes NIST data, measured data for the NSTX brazed OH conductor, and the results of fracture mechanics calculations(the wavy lines) NIST data used in this plot is shown below. The "FractMech with FS" line is wavy because for each stress level three crack area are plotted together: .353 .5, and .7 mm. The fracture mechanics calculations include factors of 2 on flaw size (so the simulations were run for .707,1, and 1.414 mm), 1.5 on fracture toughness, and 4 on cycles. To meet the required 60000 cycle life, with flaw sizes less than .7mm, 125 MPa would pass the fracture mechanics criteria in the NSTX criteria document. The criteria document makes a distinction between component and material tests for establishing the required factor of safety on life. NSTX has the three brazed conductor sample tests which show performance better than the fracture mechanics calculations. Based on the SN NIST data, 180 MPa would pass the 20 on life, but not two on stress. Approximately 112 MPa would be the allowable based on 2 on stress.

Selected Results of Fracture Mechanics Calculations - FDR calculations - See EXCEL spreadsheet copper SN Curve 2.xls and Fat2results.xls

No Safety Factor

Crack Area	c	m	k1c	wm	Width	Thickness	b/a	b	peak1	r	num of Peaks	nlife
0.125	1.52E-12	4.347	150	0.8	0.03	0.0077	1	0.000354	145	0	1	971659
0.25	1.52E-12	4.347	150	0.8	0.03	0.0077	1	0.0005	145	0	1	640022
0.5	1.52E-12	4.347	150	0.8	0.03	0.0077	1	0.000707	145	0	1	418834
1	1.52E-12	4.347	150	0.8	0.03	0.0077	1	0.001	145	0	1	270882

With Safety Factor of 2 on crack and 1.5 on Fracture Toughness

Crack Area	c	m	k1c	wm	Width	Thickness	b/a	b	peak1	r	num of Peaks	nlife
0.125	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.000707	145	0	1	418334
0.25	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.001	145	0	1	270882
0.5	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.001414	145	0	1	173083
1	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.002	145	0	1	108323
0.125	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.000707	100	0	1	2103380
0.25	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.001	100	0	1	1363689
0.5	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.001414	100	0	1	870333
1	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.002	100	0	1	544790
0.125	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.000707	50	0	1	
0.25	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.001	50	0	1	
0.5	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.001414	50	0	1	15732952
1	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.002	50	0	1	10567053
0.125	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.000707	200	0	1	103380
0.25	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.001	200	0	1	66937
0.5	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.001414	200	0	1	42770
1	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.002	200	0	1	
0.125	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.000707	250	0	1	39190
0.25	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.001	250	0	1	25375
0.5	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.001414	250	0	1	16214
1	1.52E-12	4.347	100	0.8	0.03	0.0077	1	0.002	250	0	1	

Appendix A
FAT2.FOR (written by J. Feng of MIT-PSFC, Modified slightly by P. Titus)

```
      program mastersc
c     master program for surface crack (two pulses with diff peak and r)
      common /cons/ c,rm,rkc,wm,t,w
      common /result/ af,bf,nc1,nc2
!      namelist /param/c,rm,rkc,wm,t,w
!      open(12,file='constsc.dat',status='old')
!      read(12,param)
!      close(12)
      print *, '    Fracture Mechanics Program for a Surface Crack'
      print *, '    (two pulses with different peaks and r values)'
      print *, '               Units are meters and MPa'
      print *, ' (two pulses with different peak Stresses and r values)'
      print *, 'Input:'
      print *, '  Paris constant parameters: c, rm, rkc, wm, t, w'
      print *, '    1  Enter Your Own Data'
      print *, '    2  ITER TF Case 316 Forging at 4K'
      print *, '    3  NIST 316 data'
      print *, '    4  ITER EU/KFK ICMC M2-H-03 Casting data'
      print *, '    5  C=9.54e-11, m=2.09'
      print *, '    6  C=5.43e-12, m=2.95'
      print *, '    7  C=4.41e-11, m=2.25'
      print *, '    8  Hardened Copper'
      print *, '    Enter option number:'
      read(5,*) nopt
      print *, '  Initial crack Ratio and Width: b_ai=(b/a)i, bi'
      print *, '  Pulse 1: peak1, r1 and n1 cycles/per repeat'
      print *, '  Pulse 2: peak2, r2 and n2 cycles/per repeat'
      print *, 'Output:'
      print *, '  final crack: af,bf'
      print *, '  nc1 pulse 1, nc2 pulse 2 (in the finalrepeat)'
      print *, '                    -'

      if (nopt.eq.1) then
      print *,'Enter: c,rm,rkc,wm,t,w'
      read(5,*) c,rm,rkc,wm,t,w
      end if
      if (nopt.eq.2) then
      c=6.65e-13
      rm=3.34
      rkc=200.
      wm=0.64
      t=0.1
      w=1.0
      end if
      if (nopt.eq.3) then
      c=(9.54e-11+5.43e-12+4.42e-11)/3
```

```
      c=(4.8398e-12+5.43e-12)/2
      c=5.43e-12
      rm=(2.09+2.95+2.25)/3
      rm=2.95
      rkc=100.
      wm=0.64
      t=0.0190500
      w=1.0
      end if

      if (nopt.eq.4)  c=6.619e-14
      if (nopt.eq.4)  rm=3.856
      if (nopt.eq.5)  c=9.54e-11
      if (nopt.eq.5)  rm=2.09
      if (nopt.eq.6)  c=5.43e-12
      if (nopt.eq.6)  rm=2.95
      if (nopt.eq.7)  c=4.41e-11
      if (nopt.eq.7)  rm=2.25

      rkc=100.0
      wm=0.64
      t=0.0190500
      w=1.0
      if (nopt.eq.8) then
cThe Paris parameters for the alloy CuCrZr are not available so far.  However, for time being,
can approximate data is adopted from a hardened copper alloy with similar yielding strength. [5]
c The Walker's coef representing load ratio effect is estimated from several load ratio test results
cfrom a hardened copper alloy.[5]

cParis parameters Cinm/cycle:
      C=1.52e-12
          rm=4.347
cWalker's coef:
      w=0.8
          end if

      print*, 'Enter: b_ai,bi,peak1,r1,n1,peak2,r2,n2'
          print*, '  Stresses in MPa and bi (initial crack) in meter'
      read(5,*) b_ai,bi,peak1,r1,n1,peak2,r2,n2
      print*, b_ai,bi,peak1,r1,n1,peak2,r2,n2
      print *, 'Paris constant parameters: c, rm, rkc, wm, t, w'
      print*,c,rm,rkc,wm,t,w
      print*, 'c=',c
      print*, 'exponent m=',rm
      print*, 'fracture toughness, K1c',rkc
      print*, 'Walker coefficient, wm',wm
      print*, 'Center Crack panel thickness t and width w, t,w=',t,w
      call sclife(b_ai,bi,peak1,r1,n1,peak2,r2,n2,nlife)
      print*, 'Initial b/a:                  ',b_ai
      ai=bi/b_ai
      print*, 'Initial crack Dimensions,ai,bi:',ai,bi
```

```
      print*, 'Final crack Dimensions,af,bf:  ',af,bf
      print*, 'nc1, nc2',nc1,nc2
      print*, 'nlife:',nlife
c       print*, af,bf,nc1,nc2,nlife
      stop
      end


      subroutine sclife(b_ai,bi,peak1,r1,n1,peak2,r2,n2,nlife)
c  code for surface crack loaded by two pulses with diff peaks and R
c  **** terminology ****
c   V~~~~~~V~~~~~~V~~~~~~V~~~~~~V~~,n1=1,n2=6,nc1=5,nc2=2,nlife=31
c  initial crack: b_ai=(b/a)i, bi
c  pulse 1: peak1, r1 and n1 cycles/per repeat
c  pulse 2: peak2, r2 and n2 cycles/per repeat
c  final crack: af,bf, nc1 pulse 1, nc2 pulse 2 (in the finalrepeat)
c  constant parameters: c, rm, rkc, wm, t, w
c
      common /cons/ c,rm,rkc,wm,t,w
      common /result/ af,bf,nc1,nc2
      b1=bi
      a1=bi/b_ai
      da1=0.
      db1=0.
      nc1=0
      nc=0
 1     do 21 i=1,n1
        call sc_an(b1,a1,db1,da1,peak1,r1,b2,a2,db2,da2,rka,rkb)
        b1=b2
        a1=a2
        da1=da2
        db1=db2
        nc1=nc1+1
        nc=nc+1
        if (b2.gt.t) goto 101
        if ((rkb.gt.rkc).or.(rka.gt.rkc)) goto 101
 21     continue
      nc2=0
      do 22 i=1,n2
        call sc_an(b1,a1,db1,da1,peak2,r2,b2,a2,db2,da2,rka,rkb)
        b1=b2
        a1=a2
        da1=da2
        db1=db2
        nc2=nc2+1
        nc=nc+1
        if (b2.gt.t) goto 101
        if ((rkb.gt.rkc).or.(rka.gt.rkc)) goto 101
 22     continue
      goto 1
c
```

```
c  final fracture
 101     nlife=nc
      bf=b2
        af=a2
      b_a=bf/af
      return
      end
c
c
      subroutine sc_an(bi,ai,dbi,dai,st,r,bf,af,dbf,daf,rka,rkb)
c      crack growth of sc_an per cycle
      common /cons/ c,rm,rkc,wm,t,w
      pi=3.14159
      b=bi
      a=ai
      da=dai
      db=dbi
c
 1       if (a.lt.w) then
        atemp=a+da/2
        btemp=b+db/2
          call surface_a(atemp,btemp,t,w,yatemp)
            call surface_b(atemp,btemp,t,w,ybtemp)
          rkatemp=yatemp*st*sqrt(pi*btemp)
        rkbtemp=ybtemp*st*sqrt(pi*btemp)
          dkatemp=rkatemp*(1.-r)**wm
        dkbtemp=rkbtemp*(1.-r)**wm
        dadn=c*dkatemp**rm
        dbdn=c*dkbtemp**rm
        da=dadn
        db=dbdn
        a=a+da
        b=b+db
            call surface_a(a,b,t,w,ya)
            call surface_b(a,b,t,w,yb)
        rka=ya*st*sqrt(pi*b)
        rkb=yb*st*sqrt(pi*b)
      else
        btemp=b+db/2
        call sen(btemp,t,ybtemp)
        rkbtemp=ybtemp*st*sqrt(pi*btemp)
        dkbtemp=rkbtemp*(1.-r)**wm
        dbdn=c*dkbtemp**rm
        db=dbdn
        b=b+db
        call sen(b,t,yb)
            rkb=yb*st*sqrt(pi*b)
      endif
      daf=da
      dbf=db
      af=a
```

```
      bf=b
      return
      end
c
c
      subroutine surface_b(a,b,tt,w,yf)
      pi=3.141593
      if (b.le.a) then
        rm1=1.13-0.09*(b/a)
        rm2=-0.54+0.89/(0.2+b/a)
        rm3=0.5-1/(0.65+b/a)+14*(1.-b/a)**24
        g=1.
        fphi=1.
        fw=sqrt(1/cos(pi*a*sqrt(b/tt)/(2*w)))
        fs=(rm1+rm2*(b/tt)**2+rm3*(b/tt)**4)*g*fphi*fw
        ek=sqrt(1.+1.464*(b/a)**1.65)
        yf=fs/ek
        else
        rm1=sqrt(a/b)*(1.+0.04*a/b)
        rm2=0.2*(a/b)**4
        rm3=-0.11*(a/b)**4
        g=1.
        fphi=sqrt(a/b)
        fw=sqrt(1/cos(pi*a*sqrt(b/tt)/(2*w)))
        fs=(rm1+rm2*(b/tt)**2+rm3*(b/tt)**4)*g*fphi*fw
        ek=sqrt(1.+1.464*(a/b)**1.65)
        yf=fs/ek
      endif
      return
      end
c
c
      subroutine surface_a(a,b,tt,w,yf)
c c     Y factor at edge points (10/13/94)
      pi=3.141593
      if (b.le.a) then
        rm1=1.13-0.09*(b/a)
        rm2=-0.54+0.89/(0.2+b/a)
        rm3=0.5-1/(0.65+b/a)+14*(1.-b/a)**24
        g=1.+(0.1+0.35*(b/tt)**2)
        fphi=sqrt(b/a)
        fw=sqrt(1/cos(pi*a*sqrt(b/tt)/(2*w)))
        fs=(rm1+rm2*(b/tt)**2+rm3*(b/tt)**4)*g*fphi*fw
        ek=sqrt(1.+1.464*(b/a)**1.65)
        yf=fs/ek
        else
        rm1=sqrt(a/b)*(1.+0.04*a/b)
        rm2=0.2*(a/b)**4
        rm3=-0.11*(a/b)**4
        g=1.+(0.1+0.35*(a/b)*(b/tt)**2)
        fphi=1.
```

```
        fw=sqrt(1/cos(pi*a*sqrt(b/tt)/(2*w)))
        fs=(rm1+rm2*(b/tt)**2+rm3*(b/tt)**4)*g*fphi*fw
        ek=sqrt(1.+1.464*(a/b)**1.65)
        yf=fs/ek
      endif
      return
      end
c
c
      subroutine sen(b,tt,yf)
      pi=3.141593
      if (b.ge.tt) b=0.999999*tt
      temp=0.5*pi*b/tt
      yf1=sqrt(tan(temp)/temp)
      yf2=0.752+2.02*b/tt+0.37*(1.-sin(temp))**3
      yf3=cos(temp)
      yf=yf1*yf2/yf3
      return
      end
```

Appendix B

Note

Date: 12/21/2009
To: Peter Titus
From: Jun Feng
Subject: Fatigue life of VS coil made of pure copper C11000

Introduction

There is a great life margin for VS coil if CuCrZr is applied for VS coil. Therefore, Titus suggested to use less expensive material, e.g. pure copper. Meanwhile, the required total life cycles for the VS coil decreases to 30,000 cycles - the same as for the total number of shots.

The following sections report the estimation data. The OFHC copper (C10100 to C10700) is very similar to C11000.

Material

Pure copper (C11000), electrolytic tough-pitch copper
99.96% Cu, 0.04% O

Mechanical Properties

Tensile (ksi)	Yielding (ksi)	Elongation (%)
32-66	10-53	4-55

* depending on: cold work, grain size, temperature etc.

Paris parameter: C=1.32e-11 m/cycles, m=3.54 [1,2];
Fracture toughness is assumed to be no less than $K_{1c} = 150 MPa\sqrt{m}$;
Walker's coef: 0.8.

Sample geometry

Width: 50mm (assumed)
Thickness: 8.75mm

Load history

Case 1: residual stress is removed during post-heat treatment

VS coil: each machine pulse includes: 10 large stress cycle and 100 small stress cycle
Large stress cycle from 55 to 75 MPa ,
Small stress cycle from 55 to 60 MPa .

Case 2: residual stress remains large about 0.5 yield strength (~25MPa)

VS coil: each machine pulse includes: 10 large stress cycle and 100 small stress cycle
Large stress cycle from 80 to 100 MPa ,

Small stress cycle from 80 to 85 MPa .

Crack configuration

Surface crack at the edge of the hole;
Initial crack dimension: 0.25mm^2, 0.5mm^2;
Initial aspect ratio: 0.2

Safety factor

Crack size: 2;
Fracture toughness: 1.5.

Results of fatigue crack growth life

Residual stress	Initial crack size (mm^2)	
	0.25	0.5
No	1e7 cycles	6.3e6 cycles
Applied	7.3e6 cycles	5.1e6 cycles

Conclusion

Pure copper can be used to replace CuCrZr for VS coils.

However, it is noted that work hardening can increase copper fatigue resistance, but the water environment, higher temperature and irradiation can decrease its fatigue resistance.

References

[1] N.J. Simon and R.P. Reed, "Cryogenic properties of copper and copper alloy," NBS, DOE, 1987.
[2] N.J. Simon, E.S. Drexler, and R.P. Reed, "Properties of copper and copper alloys at cryogenic temperature," NIST Monograph 177, 1992.

Appendix C

Note

Date: 11/24/2009
To: Peter Titus
From: Jun Feng
Subject: Fatigue life of NSTX conductor

Material
Hardened copper;
Paris parameter: C=1.52e-12 m/cycles, m=4.347 ;
Fracture toughness : $K_{1c} = 150 MPa\sqrt{m}$ **;**
Walker's coef: 0.8.

Sample geometry
Width: 30mm (assumed)
Thickness: 7.7mm

Load history
0 to 149 MPa along axial direction.
Stress gradient at the hole edge is neglected.

Crack configuration
Surface crack at the edge of the hole;
Initial crack dimension: 0.25mm^2, 0.5mm^2;
Initial aspect ratio: 1.

Safety factor
Crack size: 2;
Fracture toughness: 1.5.

Results of fatigue crack growth life

Safety factor	Initial crack size (mm^2)	
	0.25	0.5
Safety Fact Not Applied	701,000	446,000
Safety Factor Applied	446,000	277,000